ESTIMATING OF SHALLOW VELOCITY STRUCTURE BY INVERSION OF RECEIVER FUNCTION

Anatoly PETUKHIN¹ and Ken MIYAKOSHI²

SUMMARY

The receiver function method is widely used to estimate Earth crust and mantle structure. But to apply it to the estimation of shallow crust structure (1-5 km depth) or depth of sedimentary layer, which are important for prediction of strong ground motions, someone need to calculate receiver function at high frequencies, ~3-5 Hz. High-frequency seismic waves are strongly scattered and calculation of receiver function in most cases is impossible. To avoid this problem, we can use close earthquakes, e.g. aftershocks, or a deep subduction earthquakes. Another problem, which is general problem for the velocity structure inversions, is the trade-off between velocity in a layer and thickness of layer. To resolve this trade-off, we can apply constraints to velocity in the layer, e.g., based on the sonic logging in a neighboring borehole, and estimate only thickness of the layer. We applied developed methodology to estimate shallow structures at the K-net sites in the region of M6.1, Yamaguchi 1997 Earthquake in Japan. Structure of layers were fixed according to the geological structure in the region, velocity at shallowest layer was fixed using data from K-net sites, velocity of deepest layer was fixed according to the crustal structure, velocities in the intermediate layers were constrained using sonic logging results at a neighboring borehole Kik-net sites that penetrate into the same geological layers. Then, receiver functions were inverted for velocity structure using Genetic Algorithm; propagator matrix algorithm was used to calculate theoretical receiver functions. To validate estimated velocity structures we calculated synthetic waveforms for each site, using the discrete wavenumber method, and compared them with the observed waveforms in frequency range 0.5 - 2Hz.

1. INTRODUCTION

Since the pioneering work of [Langston, 1979] receiver function (hereafter we will refer it as RF) method is widely used to estimate Earth crust and mantle structure. RF is approach helpful to remove effects of source and path by deconvolution of the vertical component from the radial component. In its straightforward application RF is used to detect time delay of the Ps converted phases and then depth of the interfaces are estimated using a fixed velocity values in the layers. With the development of the nonlinear inversion technique (e.g., Genetic Algorithm or Neighborhood Algorithm), and with increasing of computer power, direct inversion of the RF into the velocity structure becomes popular (e.g., Shibutani et al., 1996).

For the strong ground motion simulation, knowing of the shallow velocity structure, i.e., upper few kilometers of the Crust is a matter of grate importance. Amplification of seismic waves occurs in these layers, as well as in the sedimentary layers. In order to apply the RF method to the estimation of the structure of a thin layers in this case, someone need to calculate receiver function at high frequencies, ~ 3-5 Hz. There are a few problems when someone tries to apply the RF method in high-frequency range. First, velocity structure in the high-frequency range is far from the one-dimensional structure (1-D), which is necessary to assume for the RF inversion. Second,
due to scattering effect, the input $P$-waves comes from different directions, especially in the later part, making estimation of the multiple reflected/converted phases impossible. Third, ellipse of the particle motion can differ from the direction to source, or become not an ellipse at all. All these problems restrict application of the method.

There are several other problems, where the knowing of shallow structure is proved to be useful. E.g., source inversion, reference site effect estimation and nonlinear ground response estimation. For the source inversion, knowing of shallow velocity structure under the sites allows to simulate waveforms and get slip distribution respectively in the frequency range higher than usually assumed 0.5-1Hz. Reference site effect should be assumed in the spectral inversion method to resolve trade-off between source and site effects. RF method gives reasonable estimate in this case. In order to estimate nonlinear effect of ground, some reference velocity model should be estimated before.

In this paper we propose approach to the selection of suitable data, calculation and inversion of the RF. The method was applied to a set of K-net strong-motion sites in the region of the 1997 Yamaguchi earthquake, Japan (see Figure 1), and estimated structures were validated by the waveform calculation. The results were used for the source inversion in the intermediate frequency range [Miyakoshi et al., 2004].

Figure 1: Map of studied area in Yamaguchi, Shimane and Hiroshima prefectures. Circles – target sites, stars – aftershocks of the 1997 Yamaguchi earthquake, M6.1, used in this study.

2. METHOD

In order to estimate shallow velocity structure we use nonlinear global search inversion of the observed high-frequency RFs. In this section we discuss measures that should be taken to avoid 3-D effects on the observed RF.

2.1 Data Selection Criteria

Selection of proper data for analysis is first important step for the calculation of observed high-frequency RFs. There are two problems.

1. Effect of scattering in the heterogeneous velocity structure. Due to this effect $P$-waveform becomes complex random process. Interpretation of some amplitude phases as converted phases becomes questionable. Moreover, due to scattering, incident angle for different segments of record differ from the incident angle of the direct wave, while for the calculation of the synthetic RF we should assume
existence of the direct wave only. To reduce effect of scattering we can select short distance records only. In this case effect of scattering is smaller both due to shorter travel distance, and due to smaller reflections from the reflective lower crust, which is inherent in the continental type crust, like in Western Japan. Another alternative is to use deep earthquakes. In this case, effect of scattering is smaller due to the subvertical incidence of waves to the lower crust reflectors.

2. Effect of the source mechanism. For the rays in horizontal plane, coming in the nodal direction, direct P-wave become small compare to the scattered waves. Due to this, converted waves become undetectable in the scattered noise-like signal. Contrary, for rays coming at a large angle to the nodal direction, direct P-wave amplitude becomes larger than the scattered waves and converted phases are easily detectable in the noise.

Taking these two problems into account we propose to select data for RF analysis following next criteria: (1) records of the upper crust events having hypocentral distance $R < R_{max} = 30-50km$; (2) records of the deep events (e.g. subduction zone earthquakes) having incident angle 15-45°; (3) records at the stations out of nodal planes of earthquake. Generally, visual analysis of records is enough; records that have large direct wave and relatively small P-coda, can be accepted for analysis.

2.2 Calculation of the Observed RF

To calculate the observed RF, similar to [Langston, 1979] we used classical frequency domain source equalization method. In the high-frequency range there are some specific problems listed below.

2.2.1 Selection of time window for analysis

Segment of the P-wave for analysis should start far before the P-arrival. This insures enough data for the estimation of noise level. From another side, the P-wave segment should finish at $t_{min}$ time not later than half of the S-P time. In this case segment will be free from the Sp converted wave.

Last criterion also helps to estimate minimum hypocentral distance of the record. According to the criterion we have:

$$t_{min} < 0.5t_{S-P} = 0.5R \frac{V_P}{V_S} \frac{V_P^{path} - 1}{V_P},$$

where $V_P/V_S^{path}$ is the average ratio for the propagation path. From another side, to detect reliably Ps phase converted on the deepest interface, we should follow another criterion:

$$t_{min} > H \left( \frac{1}{V_S} - \frac{1}{V_P} \right) = H \frac{V_P}{V_S^{site}} \frac{V_S^{site} - 1}{V_P},$$

where $V_P/V_S^{site}$ is the average ratio for the shallow structure and $H$ is depth of deepest interface. Combining both criteria, the minimal hypocenter distance $R_{min}$ can be estimated as:

$$R_{min} > 2H \frac{V_P}{V_S^{site}} \frac{V_S^{site} - 1}{V_P/V_S^{path} - 1}.$$  

Assuming that $V_P/V_S^{path} = 1.73$ and $V_P/V_S^{site} = 1.73$ for the hard rock sites, 2.1 for rock sites, and 2.4 for the sedimentary valley sites, and assuming that $H = 5km$, we have cautious estimates $R_{min} = 10, 15$ and 20km respectively. Except for the stations in the middle of dense earthquake swarms, this criterion usually doesn’t restrict data available for analysis.
2.2.2 Rotation to the radial and transverse components

Rotation of record components to the radial and transverse component is important step in the RF analysis, because synthetic RFs are calculated for the 1-D velocity structures, that suppose that $P_s$ converted phases should appear in the radial component only. In high frequency range, azimuth of the radial motion can be different from the azimuth to the source. This becomes possible due to some 3-D effects, for example refraction on an inclined interface in the vicinity of site, e.g., bedrock wall of the valley.

In order to rotate record components to the radial and transverse components we applied next procedure.

1. Low-pass filtering of the record, $f < 3$ Hz.
2. Rotation to the azimuth to source.
3. Plot of the particle motion and estimation of the correction angle.
4. Additional rotation to the correction angle.

Example of the particle motion and estimated correction angle is shown in the Figure 2.

![Figure 2: Example of the azimuth correction estimation, using particle motion. Horizontal components are rotated to the azimuth to source, before plotting the particle motion. Cross – start of $P$-wave, dashed line – proposed azimuth correction, -11° in this case.](image)

2.2.3 Selection of the frequency range

Usually, quasi-layered 1-D velocity model is acceptable in the low-frequency range, but it gradually changes to the 3-D velocity model with increasing of the analysed frequency. In this sense, selection of low-frequency range is appropriate for the 1-D RF analysis. From another side, increasing of the analysed frequencies reduces width of the direct and converted phases and improves resolution of inversion. To select optimal high-frequency cutoff $f_{cut}$ we used the fact that with increasing of frequency, 3-D scattering effects increase spectral amplitudes of the transverse component $A_{trans}$ and gradually they become similar to the amplitudes of the radial component, $A_{rad}$.

Using the plot of amplitude spectra of the radial and transverse components we selected $f_{cut}$ in such way that below $f_{cut}$: $A_{trans} < A_{rad}$, but above $f_{cut}$: $A_{trans} \sim A_{rad}$. Figure 3 shows example of this procedure.

Application of the above procedure to real data shows, that $f_{cut}$ varies in wide range, from 1.5Hz to 8 Hz. Sites having $f_{cut} = 1.5$Hz are practically useless for the shallow velocity structure estimation. Sites having $f_{cut} = 8$Hz promise good resolution, but those are mostly hard rock sites, for which velocity structure is very simple. Target sites in our study have $f_{cut} = 3$-8Hz. We assumed $f_{cut} = 4$Hz for all target sites, which is 2 times larger than the highest frequency of the source inversion, 2Hz [Miyakoshi et al., 2004].

Figure 4 shows example of calculation of the observed RF: for radial, transverse and vertical components. The figure shows both RFs of individual records and their average for the radial component. RF of the transverse component is useful to estimate acceptable noise level: average radial RF should be cut out at the moment when it is become comparable with the average transverse RF. Note small noise values in this example that are indicated by smaller amplitudes in the transverse RF and in the segment before the $P$-arrival. Vertical RF was calculated for reference, it clearly indicates location of the $P$-arrival and polarity of RFs after deconvolution.
Figure 3: Example of the $f_{\text{cut}}$ estimation. Arrow shows estimated value, $f_{\text{cut}} = 5\text{Hz}$ in this case. Below the $f_{\text{cut}}$, amplitudes of the radial component are larger than amplitudes of the transverse component, while both are almost the same above the $f_{\text{cut}}$ frequency.

Figure 4: Example of the observed RF calculation. Thin solid lines – RFs for individual records, thick dashed line on the upper plot – average radial RF. Note small noise values, that are indicated by smaller amplitudes in the transverse RF and in the segment before the $P$-arrival. Vertical RF was calculated for reference, it clearly indicates location of the $P$-arrival and polarity of RFs after deconvolution.
2.3 Calculation of the Synthetic RF

For calculation of the synthetic RF we developed original program using propagator matrix algorithm of [Kennett and Kerry, 1979]. The program calculates complex radial and vertical spectral responses at the top of pack of velocity layers for $P$-wave $\delta$-pulse, incident at the bottom of the pack. Then, synthetic RF is calculated by the spectral deconvolution.

3. INVERSION

For nonlinear inversion we used next three-step methodology. First step is random search. 2000 models were generated randomly and subset of 500 best models was selected as the initial population for next step. Second step is Genetic Algorithm search. Fitting of the narrow band wavelet-type RF is essentially multi-minimum problem. In order to ensure penetration from local minima to the global minimum, we set relatively large probability, 0.25, for the keeping of bad models in the tournament selection. Disadvantage of this approach is that it reduces convergence to the bottom of the global minimum. In order to improve convergence we applied third step – simplex search of the minimum of the object function, assuming that best model from the second step is located inside the global minimum valley.

There is well-known trade-off between velocity in the layer and thickness of layer that usually cannot be resolved: simultaneous increasing of the velocity and decreasing of thickness produces the same time delay of converted $Ps$ phase. To resolve this trade-off, and also to reduce number of unknown parameters of inversion we applied several constraints on the model parameters.

3.1 Geological Constraints

Traditional approach to resolve the velocity-thickness trade-off is to divide model into layers according to the geological structure under site, estimated from the borehole or by seismic reflection profiling in the studied region. Then, velocities in the layers are fixed based on the borehole sonic logging. In this case sonic logging can be done only at one point somewhere in the middle of group of target sites. Disadvantage of this approach is that deep borehole, penetrating all layers is necessary.

We used alternative approach, in which sonic logs are made at several shallow boreholes (Kik-net strong-motion sites), at the outcrops of the layers. Figure 5 shows geological map of studied region. Velocity structure can be divided into 3 layers: uppermost sedimentary layer, volcanic rocks and old rocks.

In order to constrain velocity structure, we applied next procedure.

1. For each target K-net site we looked for a nearby Kik-net site having the same geological environment. Such pairs were found for 5 sites: HRS004 - HRSH10, SMN011 - SMNH09, SMN013 - SMNH07, YMG001 - YMGH09 and YMG002 - YMGH14.

2. Using Kik-net sonic log results, we estimated average velocities $V_P$ and $V_P/\sqrt{V_S}$ ratios for the volcanic rock layer, 3900$m/s$ and 1.73 respectively, and the old rock layer, 4600$m/s$ and 1.89 respectively. Values 1.73 and 1.89 well agree with the average estimations for the extrusive and metamorphic rocks in Japan [The Working Committee, 1988].

3. Uppermost layer was constrained using sonic log results down to the 20m depths, which are available for all target sites. Lowermost structure was constrained using the regional velocity profile: 5600$m/s$ for the low-velocity layer and 6000$m/s$ for the upper crust.

4. In the intermediate depth range, we inserted one additional layer in case of the old rock site, or two additional layers in case of the volcanic rock sites. In case if the Kik-net pair site was available, $V_P$ and $V_P/\sqrt{V_S}$ for upper layer were constrained using Kik-net log data at the bottom of borehole. In case if such site wasn’t available, velocity in the upper layer, and velocity in the lower layer for all sites, were constrained using the average values for the layers, estimated at step 2.

Results of step 2 shows, that classifying sites into volcanic rock and old rock gives rather stable average velocities: $V_P = 3900_{3600}^{4300}$ m/s and $V_P = 4600_{4500}^{4700}$ m/s respectively.
3.2 Other Constraints

Density in the layers was calculated using nonlinear empirical relationship of [Ludwig at al., 1970]. $Q$-values were assumed proportional to velocities: $Q_P = 60/2000V_P$ and $Q_S = Q_P V_S / V_P$.  

Figure 5: Geological map of the studied region. Thick line bounds “outcrops” of the old rocks (Triassic and older). The rest area is covered by the volcanic rocks (Jurassic and younger). Circles – target K-net sites, triangles – Kik-net sites, having deep sonic logs down to 100m, double arrows – pairs of the nearby K-net and Kik-net sites, located in the same geological conditions, which were used to constrain of velocity models.

4. RESULTS OF INVERSION

Results of the nonlinear RF inversion were slightly corrected manually to get better waveform fit (see below). Figure 6 shows the final results for the RFs and Table 1 shows final velocity models.

Generally, velocity models in Table 1 follow to the applied geological constrains with a few exceptions. Thickness of the “old rock” layer degenerates to zero at HRS004 and SMN011 sites. Thickness of the “volcanic rock” layer degenerates to zero at YMG002 site. Additional “volcanic rock” layer appear at SMN012 site. Close analysis of these sites shows, that site YMG002 is located in the very old Earlier Cretaceous volcanic rock; while at the SMN012 site old rocks are covered by a small patch of the volcanic rock. This agrees well with the assumption that the elder rocks, the more compact and dense they are, the higher seismic velocity they have.

At the YMG007 site the upper crust layer ($V_P = 6000$ m/s) comes very close to the surface. This model can be explained by the fact that this site is located on the very old limestone outcrop (shown by blue in Figure 5, Early Carboniferous). According to [Hattori and Sugimoto, 1975] $V_p = 6000$ m/s is highly probable for limestone (~40% of measurements).
Figure 6: Results of the RF inversion. Dashed – observed RFs, solid – synthetic RFs.

Table 1: Estimated shallow velocity models.

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5. WAVEFORM CHECK

Because final target of our study was the simulation of the $S$-waveforms in the intermediate frequency range $f < 2$ Hz, for the source moment distribution inversion [Miyakoshi et al., 2004], we performed waveform check of the developed models. In order to do this we calculated waveforms using the discrete-wavenumber method [Bouchon, 1981], the same method as for the source inversion. Two aftershocks on June 25, 18h58m and on June 26, 11h45m, that have recordings at the majority of sites and the source mechanism determination at the same time, were selected. Results are shown in Figure 7. Waveform fit is good for all sites, except of YMG001. This site is in small sedimentary valley and 3-D effects disturb waveforms, especially in the later part. Unfortunately there were no records of aftershocks at YMG007, which have CMT solution necessary for the waveform calculation.

Figure 7: Comparison of the observed and synthetic $S$-waves, frequency range is 0.5 – 2 Hz.
6. DISCUSSION AND CONCLUSIONS

In this study we traditionally rotated components into the RTV coordinate system. Additional rotation into the ray coordinate system in case of the inclined incidence of the waves could increase amplitude of the $Ps$ phase and decrease $P$-wave contamination in the RF. The last can be seen in the radial RF as the large pulse near zero time, see Figure 4. In practice, in high-frequency range it is difficult to do this for several reasons: (1) incident angle depends on frequency, (2) incident angle depends on the unknown velocity structure under site and cannot be calculated theoretically, (3) in case of very shallow interface, separation of the direct $P$-phase and $Ps$ phase is impossible in a limited frequency range and estimation of the incident angle from the particle motion of the record itself is shifted to a higher value, (4) for calculation of the synthetic RF, setting of the incident angle for the output motion at the top of velocity layers is nontrivial problem comparing to the setting of the incident angle for the input motion at the bottom of the velocity layers. Actually, in case of a soft rock or sedimentary site, the incident angle is close to zero. In this study we skipped rotation into the ray coordinate system, both in the observed and synthetic RF. This procedure generates approximately the same disturbances both in the observed and synthetic RF, and inversion result should be correct.

Related to the previous problem is the problem of normalization of RF. As could be seen in Figure 6, we used normalization to the unity. This was done to reduce effect of incident angle for synthetic RF calculation: with increasing of the incident angle absolute amplitude of RF increases, while its shape is almost the same.

To get reliable information only and to reduce effects of trade-offs in the over-parameterised model, in this study we employed principle of minimum model. I.e., we prefer models with a few layers only, if it was enough to explain observed waveforms, rather than a multi-layer model. As result, fit of some receiver functions looks not good, while waveform fit is acceptable, e.g. sites SMN011 and SMN013. We should mention that, assuming that aftershocks have source mechanisms similar to the mainshock, SMN013 is located in the nodal plane of sources of aftershocks, and observed RF is badly defined in this case.

We can conclude that the RF inversion method is suitable for the estimation of shallow velocity structure for the strong ground motion simulations. Application of method is limited to sites and frequency range with the valid 1-D approximation. Waveform check is necessary for the validation and further tuning of the inverted velocity models.

Acknowledgements. This study was performed through Special Coordination Funds, titled 'Study on the master model for strong ground motion prediction toward earthquake disaster mitigation', of the Ministry of Education, Culture, Sports, Science and Technology of the Japanese Government. We also deeply appreciate National Institute of Earth Science and Disaster Prevention, for providing the K-net and Kik-net data. Many thanks to Prof. T.Shibutani for the useful consultation on the receiver function method.

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